

Equations of the line

Let $(x_1, y_1), (x_2, y_2)$ be two points in the plane. Then, the slope of the line passing through the two points is given by

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Forms:

Standard

$Ax + By = C$ where A is non-negative and A, B and C are integers.

Point – Slope

$y - y_1 = m(x - x_1)$ where m is the slope given above with the point (x_1, y_1)

Slope – Intercept

$y = mx + b$ where m is the slope given above and the y -intercept of b .

The y -intercept is the point where the line crosses the y -axis (or vertical axis) and the x -intercept is point where the line crosses the x -axis.

Example: Let $(-1, 2)$ and $(3, 6)$ be two points. Then, $x_1 = -1$ and $y_1 = 2$. And, $x_2 = 3$ and $y_2 = 6$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{-1 - 3} = \frac{-4}{-4} = 1$$

So, using the slope $m = 1$ and the point $(-1, 2)$ we have,

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = 1(x - (-1)) \Rightarrow y - 2 = 1(x + 1) \text{ giving us the point slope form and}$$

$$y - 2 = 1(x + 1) \Rightarrow y - 2 = x + 1 \Rightarrow y = x + 3 \text{ giving us the slope intercept form and}$$

$$y = x + 3 \Rightarrow -x + y = 3 \Rightarrow x - y = -3 \text{ giving us the standard form.}$$

Parallel Lines

The line Parallel to the line $y = mx + b$ is any line with the same slope, $y = mx + b'$.

Example: Find the line Parallel to the above line $y = x + 3$ and passing through the point $(-2, 3)$. Since $y = x + 3$ has slope $m = 1$ then using the Point Slope Form we have

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - (-2))$$

$$y - 3 = 1(x + 2)$$

$$y - 3 = x + 2$$

$$y = x + 5$$

So, the line Parallel to $y = x + 3$ passing through the point $(-2, 3)$ has the Slope Intercept Form

$$y = x + 5.$$

Perpendicular Lines

The line Perpendicular to the line $y = mx + b$ is the line $y = \frac{-1}{m}x + b'$.

Example: Find the line Perpendicular to the line $y = 2x + 5$ passing through the point $(-1, 2)$. Since the negative reciprocal of 2 is $\frac{-1}{2}$ then $y - 2 = \frac{-1}{2}(x - (-1)) \Rightarrow y - 2 = \frac{-1}{2}x - \frac{1}{2} \Rightarrow y = \frac{-1}{2}x + \frac{3}{2}$. So, the line

Perpendicular to $y = 2x + 5$ passing through the point $(-1, 2)$ is given by $y = \frac{-1}{2}x + \frac{3}{2}$.