

Completing the Square

Problem: Solve $ax^2 + bx + c = 0$ by completing the square.

Solution: We have,

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ca}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ca}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ca}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ca}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ca}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ca}}{2a}$$
 Thus we have the quadratic formula.

Divide both sides by a .

Subtract $\frac{c}{a}$ from both sides

Add the square of one half of $\frac{b}{a}$ to both sides. That is, one half of $\frac{b}{a}$ is $\frac{b}{2a}$. Squaring $\frac{b}{2a}$ gives us $\frac{b^2}{4a^2}$.

Combine the right hand side of equation. That is, since the

LCD of a and $4a^2$ is $4a^2$ then we multiply $-\frac{c}{a}$ by $\frac{4a}{4a}$ so

that $-\frac{c}{a} = -\frac{4ac}{4a^2}$.

Since $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$ forms a perfect square.

Take the square root of both sides.

Since $\sqrt{\frac{b^2 - 4ca}{4a^2}} = \frac{\sqrt{b^2 - 4ca}}{\sqrt{4a^2}}$.

Problem: Solve $2x^2 - 3x - 3 = 0$ by completing the square.

Solution: We have,

$$2x^2 - 3x - 3 = 0$$

$$x^2 - \frac{3}{2}x - \frac{3}{2} = 0$$

$$x^2 - \frac{3}{2}x = \frac{3}{2}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{3}{2} + \frac{9}{16}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{33}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{33}{16}$$

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \pm\sqrt{\frac{33}{16}}$$

$$x - \frac{3}{4} = \pm\frac{\sqrt{33}}{4}$$

$$x = \frac{3}{4} \pm \frac{\sqrt{33}}{4} = \frac{3 \pm \sqrt{33}}{4}$$

Divide both sides by 2.

Add $\frac{3}{2}$ to both sides.

Add $\frac{9}{16}$ to both sides. That is, since $\frac{3}{4}$ is one half of $\frac{3}{2}$ then we square it and add to both sides.

Combine $\frac{3}{2} + \frac{9}{16}$.

Since $x^2 - \frac{3}{2}x + \frac{9}{16}$ is a perfect square.

Take square root of both sides.

Simplify.

Add $\frac{3}{4}$ to both sides and combine.