7-3 Factoring by Grouping

Example 1: Factor $3my - ab + am - 3by$.

$$3my - ab + am - 3by =$$
$$3my - 3by + am - ab =$$
$$3y(m-b) + a(m-b) =$$
$$(3y+a)(m-b)$$

Example 2: Factor $2x^3 - 5xy^2 - 2x^2y + 5y^3$.

$$2x^3 - 5xy^2 - 2x^2y + 5y^3 =$$
$$2x^3 - 2x^2y + 5y^3 - 5xy^2 =$$
$$2x^2(x-y) + 5y^2(y-x) =$$
$$2x^2(x-y) - 5y^2(x-y) =$$
$$(2x^2 - 5y^2)(x-y)$$

#38 p288: Factor $\frac{1}{3}b^2 + 2b + 3$. First of all, we would like to have integer coefficients. Since we have a lowest common denominator, $\text{LCD}(3,1,1) = 3$ of 3 (where 3, 1, and 1 are the denominators of $\frac{1}{3}$, 2 and 3 respectively) then we multiply $\frac{1}{3}b^2 + 2b + 3$ by 3. We have,

$$\frac{1}{3}b^2 + 2b + 3 =$$
$$\frac{3}{3}\left(\frac{1}{3}b^2 + 2b + 3\right) =$$
$$\frac{1}{3}(b^2 + 6b + 9) =$$
$$\frac{1}{3}(b+3)^2$$

#39 p288: Factor $m^2 + \frac{5}{12}m - \frac{1}{6}$. Again, we find lowest common denominator, $\text{LCM}(1,12,6) = 12$ where 1, 12 and 6 are the denominators of 1, $\frac{5}{12}$, and $\frac{1}{6}$ respectively. So we multiply through by $\frac{12}{12}$. We have,

$$\begin{aligned}
&m^2 + \frac{5}{12}m - \frac{1}{6} = \\
&\frac{12}{12}\left(m^2 + \frac{5}{12}m - \frac{1}{6}\right) = \\
&\frac{1}{12}\left(12m^2 + 5m - 2\right)
\end{aligned}$$

Now we consider $12m^2 + 5m - 2$. Since $12 \cdot 2 = 24$ where 12 and 2 come from $\boxed{12}m^2 + 5m - \boxed{2}$. Now the factorizations of 24 are

$$\begin{array}{c}
24 \\ \hline
1 \cdot 24 \\
2 \cdot 12 \\
3 \cdot 8 \\
4 \cdot 6
\end{array}$$

Since 3 and 8 is the factorization with a difference of 5 then we have

$$\begin{aligned}
&12m^2 + 5m - 2 = \\
&12m^2 + 8m - 3m - 2 = \\
&\left(12m^2 + 8m\right) - \left(3m + 2\right) = \\
&4m\left(3m + 2\right) - 1\left(3m + 2\right) = \\
&\left(4m - 1\right)\left(3m + 2\right)
\end{aligned}$$

So that, $m^2 + \frac{5}{12}m - \frac{1}{6} = \frac{1}{12}\left(12m^2 + 5m - 2\right) = \frac{1}{12}\left(4m - 1\right)\left(3m + 2\right)$.

#38 p288: Factor $\frac{1}{4}x^2 + \frac{3}{2}x + 2$. We find the $\text{LCM}(4,2,1) = 4$ where 4, 2 and 1 are the denominators of $\frac{1}{4}, \frac{3}{2}$, and 2 respectively. So,

$$\begin{aligned}
&\frac{1}{4}x^2 + \frac{3}{2}x + 2 = \\
&\frac{4}{4}\left(\frac{1}{4}x^2 + \frac{3}{2}x + 2\right) = \\
&\frac{1}{4}\left(x^2 + 6x + 8\right) = \\
&\frac{1}{4}\left(x + 2\right)\left(x + 4\right)
\end{aligned}$$

7 – 5 Factoring Trinomials

Example 1: Factor $2q^2-9q-18$. Since the coefficient of q^2 is 2 and the constant is 18 then we consider the factorizations of $2\cdot 18=36$ where 2 and 18 come from $\boxed{2}q^2-9q-\boxed{18}$;

$$\begin{array}{c} 36 \\ \hline 1\cdot 36 \\ 2\cdot 18 \\ 3\cdot 12 \\ 4\cdot 9 \\ 6\cdot 6 \end{array}$$

Since 9 is negative and 18 is negative then we look for a factorization of 36 that has a difference of 9. Since the only difference of 9 in the factorizations is between 12 and 3 then we have

$$\begin{aligned} &2q^2-9q-18= \\ &2q^2+(3-12)q-18= \\ &2q^2+3q-12q-18= \\ &q(2q+3)-6(2q+3)= \\ &(q-6)(2q+3) \end{aligned}$$

Example 2: Factor $8m^2-10m+3$. Since the coefficient of m^2 and the constant is 3 then we consider the factorization of $8\cdot 3=24$ where 8 and 3 come from $\boxed{8}m^2-10m+\boxed{3}$;

$$\begin{array}{c} 24 \\ \hline 1\cdot 24 \\ 2\cdot 12 \\ 3\cdot 8 \\ 4\cdot 6 \end{array}$$

Since 10 is negative and 3 is positive in $8m^2-10m+3$ then -10 is a sum of negative numbers. Since $4\cdot 6$ has the only terms that sum to 10 then we have,

$$\begin{aligned} &8m^2-10m+3= \\ &8m^2-(4+6)m+3= \\ &8m^2-4m-6m+3= \\ &4m(2m-1)m-3(2m-1)= \\ &(4m-3)(2m-1) \end{aligned}$$

7 – 6 Factoring Differences of Squares $\boxed{a^2 - b^2 = (a+b)(a-b)}$

Example 1: Factor $45x^2 - 20y^2z^2$. Since neither 45 nor 20 are perfect squares and the $\text{GCF}\left(45x^2, 20y^2z^2\right) = 5$ then we factor 5 from $45x^2 - 20y^2z^2$. So that $45x^2 - 20y^2z^2 =$ $5\left(9x^2 - 4y^2z^2\right)$. Since $9x^2$ and $4y^2z^2$ are perfect squares then $\sqrt{9x^2} = 3x$ and $\sqrt{4y^2z^2} = 2yz$. Thus, $5\left(9x^2 - 4y^2z^2\right) = 5(3x + 2yz)(3x - 2yz)$.

Example 2: Factor $0.01n^2 - 1.69r^2$. Since $\sqrt{0.01n^2} = 0.1n$ and $\sqrt{1.69r^2} = 1.3r$ then $0.01n^2 - 1.69r^2 = (0.1n + 1.3r)(0.1n - 1.3r)$.

7 – 7 Perfect Squares and Factoring $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$ and $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

Example 1: Factor $4k^2 - 4k + 1$. Since $\sqrt{4k^2} = 2k$ and $\sqrt{1} = 1$ and $2 \cdot 2k \cdot 1 = 4k$ then $4k^2 - 4k + 1 = (2k - 1)^2$.

Example 2: Factor $9a^2 + 12a - 4$. This is not a perfect square trinomial because $+12a$ is positive and -4 is negative. Meaning we would have a product like this $(\_ + \_)(\_ - \_)$.

Example 3: Factor $\frac{4}{9}x^2 - \frac{16}{3}x + 16$. Since $\sqrt{\frac{4}{9}x^2} = \frac{2}{3}x$ and $\sqrt{16} = 4$ and $2 \cdot \frac{2}{3}x \cdot 4 = \frac{16}{3}x$ then $\frac{4}{9}x^2 - \frac{16}{3}x + 16 = \left(\frac{2}{3}x - 4\right)^2$