Secant Line

The slope of the secant line of a function is given by

$$m_{\sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

where $(x_0, f(x_0))$ and $(x_1, f(x_1))$ are points on the graph of $f(x)$.

Example: Find the slope of the secant line of $f(x) = 2x^2 + 3$ for $x_0 = 0$ and $x_1 = 1$. We have,

$$m_{\sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(1) - f(0)}{1 - 0} = f(1) - f(0) = 2(1)^2 + 3 - \left[2(0)^2 + 3\right] = 5 - 3 = 2.$$

Tangent Line

The slope of the tangent line at a point $(x_0, f(x_0))$ is given by

$$m_{\tan} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

Example: Find the slope of the tangent line $f(x) = 2x^2 + 3$ at $x_0 = 2$. We have,

$$\begin{aligned} m_{\tan} &= \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} = \lim_{h \to 0} \frac{2(2 + h)^2 + 3 - \left[2(2)^2 + 3\right]}{h} \\ &= \lim_{h \to 0} \frac{2(4 + 4h + h^2) + 3 - [11]}{h} = \lim_{h \to 0} \frac{8 + 8h + 2h^2 + 3 - [11]}{h} = \lim_{h \to 0} \frac{8h + 2h^2}{h} = \lim_{h \to 0} \frac{2h(4 + h)}{h} \\ &= 2 \lim_{h \to 0} (4 + h) = 2 \cdot 4 = 8 \end{aligned}$$

The equation of the tangent line at a point $(x_0, f(x_0))$ if given by

$$y - f(x_0) = m_{\tan}(x - x_0)$$

Example: From the previous example let $f(x) = 2x^2 + 3$ and $x_0 = 2$. Since $(x_0, f(x_0)) = (2, 11)$ then we have,

$$\begin{aligned} y - f(x_0) &= m_{\tan}(x - x_0) \\ y - 11 &= 8(x - 2) \\ y &= 8x - 5 \end{aligned}$$

The Derivative

The derivative of a function $f(x)$ is given by

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Example: Find the derivative of $f(x)=2x^2+3$. We have,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2(x+h)^2+3-(2x^2+3)}{h}=\lim_{h\to 0}\frac{2(x^2+2xh+h^2)+3-(2x^2+3)}{h}$$

$$=\lim_{h\to 0}\frac{2(x^2+2xh+h^2)+3-(2x^2+3)}{h}=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+3-2x^2-3}{h}$$

$$=\lim_{h\to 0}\frac{4xh+2h^2}{h}=\lim_{h\to 0}\frac{h(4x+2h)}{h}=\lim_{h\to 0}(4x+2h)=4x$$